

## Abstracts of Papers to Appear in Future Issues

**BOUNDARY CONDITIONS FOR OPEN BOUNDARIES FOR THE INCOMPRESSIBLE NAVIER–STOKES EQUATION.** B. Christer V. Johansson. *The Royal Institute of Technology, NADA-C2M2, S-100 44 Stockholm, Sweden.*

In this paper we investigate new boundary conditions for the incompressible, time-dependent Navier–Stokes equation. Especially inflow and outflow conditions are considered. The equations are linearized around a constant flow, so that we can use the Laplace–Fourier technique to investigate the strength of boundary layers at open boundaries. Such layers are unphysical, and new boundary conditions are proposed so that these boundary layers are suppressed. We also show that the boundary conditions we propose do not produce divergence. Furthermore, they give solutions that do not grow in time, as long as the forcing function in the system does not. We also discuss the numerical treatment of boundaries when fourth-order accurate finite difference operators are used to approximate spatial derivatives. Using higher order methods introduces eigensolutions with boundary layer thickness of the same order of magnitude as the grid size. These eigensolutions have to be suppressed in order to not destroy the fourth-order accuracy of the method. Numerical results for the non-linear Navier–Stokes equation, together with the new boundary conditions, are presented. These calculations confirm that the results for the linearized problem hold for the non-linear problem as well.

**A TRIANGLE BASED MIXED FINITE ELEMENT–FINITE VOLUME TECHNIQUE FOR MODELING TWO PHASE FLOW THROUGH POROUS MEDIA.** Louis J. Durlofsky. *Chevron Oil Field Research Company, P.O. Box 446, La Habra, California 90633-0446, U.S.A.*

Triangle based discretization techniques offer great advantages relative to standard finite difference methods for the modeling of flow through geometrically complex geological features. The purpose of this paper is to develop and apply a triangle based method for the modeling of two phase flow through porous formations. The formulation includes the effects of gravity, compressibility, and capillary pressure. The technique entails a triangle based mixed finite element method for solution of the variable coefficient, parabolic pressure equation, and a second-order TVD-type (total variation diminishing) finite volume scheme for solution of the essentially hyperbolic saturation equation. The method is applied to a variety of example problems and is shown to perform very well on problems involving geometric complexity coupled with heterogeneous, generally anisotropic permeability descriptions.

**LAPLACE'S EQUATION AND THE DIRICHLET–NEUMANN MAP IN MULTIPLY CONNECTED DOMAINS.** A. Greenbaum and L. Greengard. *Courant Institute of Mathematical Sciences, New York University, New York, New York 10012, U.S.A.*; G. B. McFadden. *Computing and Applied Mathematics Laboratory, National Institute of Standards and Technology, Gaithersburg, Maryland 20899, U.S.A.*

A variety of problems in material science and fluid dynamics require the solution of Laplace's equation in multiply connected domains. Integral

equation methods are natural candidates for such problems, since they discretize the boundary alone, require no special effort for free boundaries, and achieve superalgebraic convergence rates on sufficiently smooth domains in two space dimensions, regardless of shape. Current integral equation methods for the Dirichlet problem, however, require the solution of  $M$  independent problems of dimension  $N$ , where  $M$  is the number of boundary components and  $N$  is the total number of points in the discretization. In this paper, we present a new boundary integral equation approach, valid for both interior and exterior problems, which requires the solution of a single linear system of dimension  $N + M$ . We solve this system by making use of an iterative method (GMRES) combined with the fast multipole method for the rapid calculation of the necessary matrix vector products. For a two-dimensional system with 200 components and 100 points on each boundary, we gain a speedup of a factor of 100 from the new analytic formulation and a factor of 50 from the fast multipole method. The resulting scheme brings large scale calculations in extremely complex domains within practical reach.

**PROCESSING OF COMPUTED VECTOR FIELDS FOR VISUALIZATION.** Susumu Shirayama. *SofTek Systems, Inc., 2-34-5, 5F, Nozawa, Setagaya-ku, Tokyo 154, Japan.*

This paper describes several methods of visualizing the vector fields in a flow analysis. A class of computational algorithms determining the structure of vector fields are stated. These algorithms can be applied to other areas of computational physics. The first part of the paper concentrates on a formulation of the problem. Usually the objective vector fields are obtained on a discretized space. This makes it difficult to construct the computational algorithms. An appropriate interpolation method has to be chosen in order to reconstruct a continuous space. Since the reconstructed space is defined locally, a grid cell in which the solution moves must be found by the efficient algorithm. This becomes a crucial problem in three dimensions. The construction of schemes is performed on both physical and computational spaces in order to overcome such a difficulty.

**FINITE VOLUME CALCULATIONS OF SELF-SUSTAINED OSCILLATIONS IN A GROOVED CHANNEL.** J. C. F. Pereira and J. M. M. Sousa. *Instituto Superior Técnico/Technical University of Lisbon, Avenida Rovisco Pais, 1096 Lisbon Codex, Portugal.*

A numerical simulation of unsteady incompressible Navier–Stokes flow in a periodically grooved channel is performed for Reynolds numbers  $R \sim O(10^3)$ . The numerical scheme is based on finite differences approximation with an explicit quadratic Leith-type of temporal discretization. Periodic self-sustained oscillatory flow is reported to occur in the investigated flow configuration, indicating a Hopf bifurcation. The threshold for the onset of self-sustained flow oscillations is the critical Reynolds number  $950 < R_c < 1050$ . The numerical predictions are in good agreement with reported spectral element method solutions, demonstrating the accuracy and efficiency of the present method.

MODELING A NO-SLIP FLOW BOUNDARY WITH AN EXTERNAL FORCE FIELD.

D. Goldstein. *Center for Fluid Mechanics, Brown University, Providence, Rhode Island 02912, U.S.A.*; R. Handler. *Naval Research Laboratory, Center for Advanced Space Sensing, Washington, DC 20375, U.S.A.*; L. Sirovich. *Center for Fluid Mechanics, Brown University, Providence, Rhode Island 02912, U.S.A.*

A novel technique related to Peskin's immersed boundary approach is used to introduce solid surfaces into a simulated flow field. The Navier-Stokes equations permit the presence of an externally imposed body force that may vary in space and time. Forces are chosen to lie along a desired surface and to have a magnitude and direction opposing the local flow such that the flow is brought to rest at an element of the surface. For unsteady viscous flow the direct calculation of the needed force is facilitated by a feedback scheme in which the velocity is used to iteratively determine the desired value. In particular, we determine the surface body force from the relation  $\mathbf{f}(\mathbf{x}_s, t) = \alpha \int_0^t \mathbf{U}(\mathbf{x}_s, t') dt' + \beta \mathbf{U}(\mathbf{x}_s, t)$  for surface points  $\mathbf{x}_s$ , velocity  $\mathbf{U}$ , time  $t$ , and negative constants  $\alpha$  and  $\beta$ . Examples are presented which include 2D flow around cylinders, 3D turbulent channel flow where one boundary is simulated with a force field, and turbulent channel flow over a riblet-covered surface. While the new method may be applied to complex geometries on a non-Cartesian mesh, we have chosen to use a simple Cartesian grid. All simulations are done with a spectral code in a single computational domain without any mapping of the mesh.

A NUMERICAL IMPLEMENTATION OF MHD IN DIVERGENCE FORM.

Maurice H. P. M. van Putten. *Theoretical Astrophysics 130-33, Caltech, Pasadena, California 91125, U.S.A.*

A numerical implementation of fully relativistic MHD in divergence form for the purpose of shock computations is presented. We illustrate the implementation on the coplanar Riemann problem for MHD. The computations are performed using a pseudo-spectral method. Brio and Wu recently demonstrated the existence of compound waves in classical MHD. We find that compound waves persist in relativistic MHD. Two limiting cases are considered numerically: a limit in which the velocities become nonrelativistic and a limit in which the longitudinal magnetic field approaches zero. A comparison is made with the results of nonrelativistic MHD and with the singular case of zero longitudinal magnetic field.

ON AN ADAPTIVE TIME STEPPING STRATEGY FOR SOLVING NONLINEAR DIFFUSION EQUATIONS.

K. Chen, M. J. Baines, and P. K. Sweby. *Institute of Computational Fluid Dynamics, University of Reading, Whiteknights, P.O. Box 220, Reading RG6 2AX, Berkshire, England.*

A new time step selection procedure is proposed for solving nonlinear diffusion equations. It has been implemented in the ASWR finite element code of Lorenz and Svoboda for 2D semiconductor process modelling

diffusion equations. The strategy is based on equidistributing the local truncation errors of the numerical scheme. The use of B-splines for interpolation (as well as for the trial space) results in a banded and diagonally dominant matrix. The approximate inverse of such a matrix can be provided to a high degree of accuracy by another banded matrix, which in turn can be used to work out the approximate finite difference scheme corresponding to the ASWR finite element method, and further to calculate estimates of the local truncation errors of the numerical scheme. Numerical experiments on six full simulation problems arising in semiconductor process modelling have been carried out. Results show that our proposed strategy is more efficient and better conserves the total mass.

A MULTIDIMENSIONAL FLUX FUNCTION WITH APPLICATIONS TO THE EULER AND NAVIER-STOKES EQUATIONS.

Christopher L. Rumsey. *NASA Langley Research Center, Hampton, Virginia 23665, U.S.A.*; Bram van Leer and Philip L. Roe. *University of Michigan, Ann Arbor, Michigan 48109, U.S.A.*

A grid-independent approximate Riemann solver has been developed for use in both two- and three-dimensional flows governed by the Euler or Navier-Stokes equations. Fluxes on grid faces are obtained via wave decomposition. By assuming that information propagates in the velocity-difference directions, rather than in the grid-normal directions as in a standard grid-aligned solver, this flux function more appropriately interprets and hence more accurately resolves shock and shear waves when these lie oblique to the grid. The model, which describes the difference in states at each grid interface by the action of five waves, produces a significant increase in accuracy for both supersonic and subsonic first-order spatially accurate computations. Second-order computations with the grid-independent flux function are generally only worth the additional effort for flowfields whose primary structures include shear waves that lie oblique to the grid. Included in this category is the viscous flow over an airfoil in which a separated shear layer emanates from the airfoil surface at an angle to the grid. Pressure distortions which can result from misinterpretation of the oblique waves by a grid-aligned solver are essentially eliminated by the grid-independent flux function.

VORONOI POLYGONS AND POLYHEDRA.

R. E. M. Moore and I. O. Angell. *The London School of Economics, University of London, Houghton Street, London WC2A 2AE, United Kingdom.*

A new algorithm for the calculation of Voronoi polytopes in 2D and 3D Euclidean space is described. Some results for random nuclei with reciprocal boundary conditions in 2D and 3D are discussed in relation to previous findings. In 2D the number of sides distribution is identified as being close to  $F(3.5; 21)$ . Kiang's conjectures for the distributions of areas and volumes for polygonal and polyhedral tessellations respectively, are supported: the challenge of deriving his  $F$ -distributions beyond 1D still stands.